1975-03

Single-channel prediction for earth tidal tilt data

Chang, Chien-Wu

Monterey, California. Naval Postgraduate School

http://hdl.handle.net/10945/21039

SINGLE-CHANNEL PREDICTION FOR EARTH TIDAL TILT DATA

Chien-Wu Chang

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

SINGLE-CHANNEL PREDICTION
FOR
EARTH TIDAL TILT DATA

by

Chang, Chien-Wu

March 1975

Thesis Advisor: D. R. Barr

Approved for public release; distribution unlimited.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Single-Channel Prediction for Earth Tidal Tilt Data		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis March 1975
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Chang, Chien-Wu		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE March 1975
		13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The earth tidal tilt data consists of two parts: one is the theoretical part caused by the tidal potential of the moon and the sun; another one is the seismic noise part that originates from meteorological, hydrodynamic, and cultural sources. The theoretical part is completely determinable by astronomical data. The seismic noise may be considered as a stationary stochastic process over short periods of time when

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
(Page 1) S/N 0102-014-6601 |

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

there have been no great changes in the source mechanisms. Based on the above assumption, prediction and interpolation of the earth tidal tilt data can be achieved by solving the Wiener-Hopf equation. A computer program "PREDICT" is derived by using Levinson's recursive method to solve the discrete-time Wiener-Hopf equation. Test results on real data are satisfactory. Besides the earth tidal tilt data, the "PREDICT" program can also be applied to any stationary stochastic discrete-time signal.

Single-Channel Prediction
for
Earth Tidal Tilt Data

by

Chang, Chien-Wu
Captain, Chinese Navy
B.S., Chinese Naval Academy, 1951
M.S., National Chiao-Tung University, 1963

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
March 1975

ABSTRACT

The earth tidal tilt data consists of two parts: one is the theoretical part caused by the tidal potential of the moon and the sun; another one is the seismic noise part that originates from meteorological, hydrodynamic, and cultural sources. The theoretical part is completely determinable by astronomical data. The seismic noise may be considered as a stationary stochastic process over short periods of time when there have been no great changes in the source mechanisms. Based on the above assumption, prediction and interpolation of the earth tidal tilt data can be achieved by solving the Wiener-Hopf equation. A computer program "PREDICT" is derived by using Levinson's recursive method to solve the discrete-time Wiener-Hopf equation. Test results on real data are satisfactory. Besides the earth tidal tilt data, the "PREDICT" program can also be applied to any stationary stochastic discrete-time signal.

TABLE OF CONTENTS

ACKNOWLEDGMENTS

The author wishes to thank Dr. M. D. Wood for his guidance throughout the course of this thesis, as well as the assistance given by Miss N. E. King. The author also wishes to acknowledge the encouragement and help received from Professor D. R. Barr and Professor R. H. Shudde. Finally, the author wishes to express his thanks to Mrs. E. H. Hirscher and R. Lewis for her patience in typing a clean copy from a cluttered manuscript.

I. INTRODUCTION

The essential problem of earthquake prediction is to identify and monitor those physical characteristics of the failure process that uniquely herald the magnitude, location and origin time of an earthquake. Identification of these characteristics has tended to be heuristic and statistical in approach; however, evidence is steadily accumulating toward the support of a sound physical basis to earthquake prediction. Models of this failure process are becoming sufficiently detailed to effectively define the spatial and temporal character in each of the variety of geophysical measurements conducted in the epicentral region.

This paper contains the theory and applications of a single-channel predictor for tilt at tidal sensitivity. Tidal tilt is caused by the direct tidal forces of moon and sun on the earth. There are also other sources such as meteorological activity and the ocean-loading effect which can cause significant tilts exceeding the magnitude of the primary source. The primary sources are predictable and, to a lesser degree, the effects of ocean and meteorological activity are also predictable. Other factors such as effects of local earthquake activity are random in nature and cannot be determined in advance. Therefore, we divide tidal-tilt information into two classes: one is predictable, the other random. In the nomenclature of communication theory, we

call the former signals and the latter noises. However in subsequent discussion we will, in fact, use the definition that the signal we seek is random in nature and the noise is the periodic but undesired part of the time series.

A single-channel predictor in time series analysis uses past data to predict future data. The formulation of a self-predictor is basically one of designing a filter whose output in the time domain is a positive time-shifted version of the input. The prediction error is a measure of the stationarity of the time series. If the time series spectral density of equal lengths of data segments is a constant, then a filter designed from any segment of past data can also be used to predict future data from any other data segment. For the geophysical problem at hand, we derive such a filter during earthquake-free periods, because we assume that the failure process is, mathematically speaking, associated with non-stationarity in the data. The strategy is thus one of deriving a filter free of this "signal"; using that filter for subsequent incoming data, and monitoring the magnitude of the prediction error as a means of identifying onset of the failure process.

II. THEORY OF SINGLE-CHANNEL PREDICTOR

A. BASIC TERMINOLOGIES

1. Digital Signals

In order to use high-speed digital computers to analyze seismic signals, the first thing we must do is to convert the analog signals to digital signals. This involves the work to extract the values of the analog signal at discrete time points which are usually equally spaced. It is convenient to choose the time unit equal to the time-spacing Δt between two consective discrete time points so that the discrete time variable t takes on only integer values. The numerical values of the signal together with their associated discrete time points make up a digital signal. One example of a digital signal is

Signal:	$\cdots$, 1, 3, 2, 4, 3, 5, 2, 4, $\cdots$
Time t:	$\cdots$, -2, -1, 0, 1, 2, 3, 4, 5, $\cdots$,

Because we have required time t to assume consecutive integer values, we can write the above digital signal simply as

$$x = (\cdots, 1, 3, \underset{\uparrow}{2}, 4, 3, 5, 2, 4, \cdots),$$

where the arrow is used to denote the time origin t = 0 and x represents the entire signal. Under this notation, x_t

will designate the amplitude of the digital signal x at time t. Thus in the above signal $x_0 = 2$, $x_1 = 4$, and $x_{-1} = 3$.

2. Wavelets

Wavelets are a special class of digital signals. A wavelet b_t is characterized by two properties, namely,

(a) The one-sided property: A wavelet has a definite origin (or arrival) time in the sense that all values of the wavelet before its origin time are zero, that is $b_t = 0$ for $t < 0$.

(b) The stability property: A wavelet has finite energy so that it is a transient or dying-out phenomenon, that is

$$\sum_{t=0}^{\infty} b_t^2$$

is finite.

From the above two properties we know that a wavelet is a one-sided energy signal. It may damp out completely, i.e., become zero, after a certain time. This is the so-called finite-length wavelet. One example is

$(\cdots, 0, 0, 0, 5, 2, 3, 0, 0, 0, \cdots)$

where we can simply write as

$(5, 2, 3)$.

We call it a three-length wavelet. A wavelet can also be of infinite-length, that is, it damps out at infinite time. One example of infinite-length wavelet is

$(1, 1/2, 1/4, 1/8, 1/16, 1/32, 1/64, 1/128, 1/256, \cdots)$.

3. Convolution in Discrete-Time Systems

In order to find a linear system's response or output resulting from a specific input signal, we have two methods

in general use. One method is to model or characterize the linear system by means of linear differential (continuous-time systems) or difference (discrete-time systems) equations. The output of the system to any input signal is then obtained by solving the appropriate equation. Another method is based on the superposition property of linear systems, using the convolution operation between the input signal and the system's impulse response to get the output signal. Since we are only interested in treating signals in the discrete-time domain, we consider the convolution operation here only in discrete-time systems.

The convolution of two wavelets a_t and b_t is denoted by

$$c = a * b$$

and is defined by the formula

$$c_t = \sum_{m=0}^{t} a_m \, b_{t-m}.$$

To illustrate the use of this formula let us suppose

$$a = (a_0, a_1, a_2) = (1, 3, 2) \text{ and}$$

$$b = (b_0, b_1, b_2) = (2, 1, 4).$$

Using the convolution formula, we have

$$c = a * b = (c_0, c_1, c_2, c_3, c_4)$$

where

$$c_0 = a_0 b_0 = 1 \times 2 = 2$$

$$c_1 = a_0 b_1 + a_1 b_0 = 1 \times 1 + 3 \times 2 = 7$$

$$c_2 = a_0b_2 + a_1b_1 + a_2b_0 = 1 \times 4 + 3 \times 1 + 2 \times 2 = 11$$

$$c_3 = a_1b_2 + a_2b_1 = 3 \times 4 + 2 \times 1 = 14$$

$$c_4 = a_2b_2 = 2 \times 4 = 8$$

Hence

$$c = a * b = (2, 7, 11, 14, 8).$$

In general case, if a_t and b_t are not wavelet but any two digital signals, the formula for convolution operation is

$$c = a * b$$

where

$$c_t = \sum_{m=-\infty}^{\infty} a_m b_{t-m}.$$

4. The Z-Transform

In analyzing linear systems, we can use linear differential or difference equations. We can also use convolution operations. Both are called time-domain techniques since the independent parameter is time t. But we find that if we transform the time signals within our system to another form, we can often express relationships more simply than we have previously in the time-domain. We call this transform approach transform-domain techniques. One important consequence of the transform-domain approach of linear systems is that the convolution operation in the time-domain is converted to a multiplication operation in the transform-domain. There are three transforms which are important in the study of linear systems. The Laplace transform and Fourier transform are used in the continuous-time

systems. The Z-transform is appropriate for discrete-time systems. We will only consider the Z-transform here. The Z-transform of finite wavelet is the polynomial in Z whose coefficients are the coefficients of the wavelet. For example, the Z-transform of the wavelet

$$a = (a_0, a_1, a_2, \ldots, a_n)$$

is the polynomial

$$A(Z) = a_0 + a_1 Z + a_2 Z^2 + \ldots + a_n Z^n.$$

The Z-transform of an infinite wavelet is the power series in Z whose coefficients are the coefficients of the wavelet. For example, the Z-transform of the wavelet

$b = (b_0, b_1, b_2 \ldots)$ is the power series

$$B(Z) = b_0 + b_1 Z + b_2 Z^2 + \ldots.$$

The Z-transform of a general two-sided digital signal which is of infinite length in both directions will be a Laurent series in Z. For example, the Z-transform of the digital signal

$$c = (\ldots, c_{-2}, c_{-1}, c_0, c_1, c_2 \ldots)$$

is the Laurent series

$$C(Z) = \ldots + c_{-2} Z^{-2} + c_{-1} Z^{-1} + c_0 + c_1 Z + c_2 Z^2 + \ldots.$$

To show convolution operation in time-domain may be performed by multiplication operation in transform domain, let us redo our example in the last section. We have

$$c = a * b$$

where

$a = (a_0, a_1, a_2) = (1, 3, 2)$ and

$b = (b_0, b_1, b_2) = (2, 1, 4)$.

The Z-transforms of a_t and b_t are

$A(Z) = 1 + 3Z + 2Z^2$ and

$B(Z) = 2 + Z + 4Z^2$.

Multiplying A(Z) and B(Z), we obtain

$$C(Z) = A(Z)B(Z) = 2 + 7Z + 11Z^2 + 14Z^3 + 8Z^4.$$

After we transform back to time-domain, we get

$$c = (c_0, c_1, c_2, c_3, c_4) = (2, 7, 11, 14, 8)$$

which is the same result as in the last section.

5. Autocorrelation and Cross-Correlation

Autocorrelation and cross-correlation are two important concepts in studying stationary stochastic processes. For our purposes here we need only consider these concepts as applied to discrete energy signals.

The autocorrelation function of any energy signal b_t is defined as:

$$\phi_{bb}(\tau) = \sum_{t=-\infty}^{\infty} b_{t+\tau} b_t^*$$

where the integer τ is the time-shift and b_t^* is the complex-conjugate of b_t. If we only consider real-valued discrete energy signals, the formula is simply:

$$\phi_{bb}(\tau) = \sum_{t=-\infty}^{\infty} b_{t+\tau} b_t.$$

When $\tau = 0$, we have

$$\phi_{bb}(o) = \sum_{t=-\infty}^{\infty} b_t^2$$

which is equal to the energy E of the signal.

If we let $s = t + \tau$ in the formula, we have

$$\phi_{bb}(\tau) = \sum_{t=-\infty}^{\infty} b_{t+\tau} b_t = \sum_{s=-\infty}^{\infty} b_s b_{s-\tau} = \sum_{s=-\infty}^{\infty} b_{s-\tau} b_s = \phi_{bb}(-\tau).$$

Thus the autocorrelation function for a real-valued discrete energy signal is an even function of the time-shift τ.

As a generalization of the autocorrelation function, the cross-correlation function of two real-valued discrete energy signals a_t and b_t is defined as:

$$\phi_{ab}(\tau) = \sum_{t=-\infty}^{\infty} a_{t+\tau} b_t = \sum_{s=-\infty}^{\infty} a_s b_{s-\tau}.$$

From the above formula it is easily seen that:

$$\phi_{ab}(\tau) = \phi_{ba}(-\tau).$$

Also we see that the autocorrelation function $\phi_{bb}(\tau)$ is a special case of the cross-correlation function $\phi_{ab}(\tau)$ occurring when the signals a_t and b_t are the same energy signal.

6. Deconvolution

Deconvolution is the inverse operation of convolution. Consider the problem that

$$c = a * b$$

which means the discrete signal c_t is equal to the discrete signal a_t convolved with discrete signal b_t. If we are given c_t and a_t and we wish to find b_t, we need to deconvolve a_t

from c_t to get b_t. We can do the deconvolution both in time-domain and in transform-domain.

$$c_t = \sum_{m=o}^{t} a_m b_{t-m}$$

The first few terms are

$$c_o = a_o b_o$$

$$c_1 = a_o b_1 + a_1 b_o$$

$$c_2 = a_o b_2 + a_1 b_1 + a_2 b_o$$

$$\vdots$$

In the first equation, we can solve b_o, because we know both c_o and a_o. After we get b_o, we can use the second equation to solve b_1. We can continue this iterative process until we find the signal b_t.

In the transform-domain, we know the convolution operation becomes a multiplication operation. Hence the deconvolution operation becomes a division operation. Thus if $c = a * b$, then after we take the Z-transform, we have

$$C(Z) = A(Z)B(Z).$$

If we are given c_t and a_t, we know C(Z) and A(Z). We can find b_t by first solving B(Z) which is equal to C(Z) divided by A(Z). Then we can transform B(Z) back to the time-domain to get the signal b_t.

To show the procedures stated above, let us do an example. Suppose

$$c_t = (c_0, c_1, c_2, c_3) = (12, 10, 14, 6)$$

$$a_t = (a_0, a_1) = (4, 2).$$

We want to find b_t such that

$$c = a * b.$$

Taking Z-transforms of c_t and a_t, we obtain

$$C(Z) = 12 + 10Z + 14Z^2 + 6Z^3$$

$$A(Z) = 4 + 2Z$$

The Z-transform of b_t is

$$B(Z) = C(Z)/A(Z) = \frac{12 + 10Z + 14Z^2 + 6Z^3}{4 + 2Z} = 3 + Z + 3Z^2.$$

Therefore

$$b_t = (b_0, b_1, b_2) = (3, 1, 3).$$

One thing we must point out here is that when we convolve two finite-length discrete-time signals, the result is always a finite-length discrete-time signal. But when we deconvolve a finite-length signal from another finite-length signal, the results are not always of finite-length. For example, if

$$c = a * b$$

$$c_t = (c_0, c_1, c_2, c_3) = (12, 10, 14, 8)$$

$$a_t = (a_0, a_1) = (4, 2)$$

we will obtain

$$b_t = (b_0, b_1, b_2, \ldots)$$

$$= (3, 1, 3, 1/2, -1/4, 1/8, -1/16, \ldots)$$

which is of infinite-length. This fact is related to the design of least error energy shaping filters, a subject we will discuss later on.

7. Stationary and Ergodic Stochastic Processes

A stochastic process is a proces that assigns, according to a certain rule, a time function (continuous or discrete) $f(t, \zeta)$ to every outcome ζ of an experiment. It can be viewed as a function of two variables t and ζ. $f(t, \zeta)$ may represent:

(1) A family of time functions (t and ζ variables)

(2) A single time function (t variable, ζ fixed)

(3) A random variable (t fixed, ζ variable)

(4) A single number (t fixed, ζ fixed).

A stochastic process $f(t)$ (we usually omit its dependence on ζ) is stationary in the general sense if its expected value is a constant and its autocorrelation depends only on t_1-t_2:

$$E\{f(t)\} = \text{constant} \quad (\text{ for any time } t)$$

$$E\{f(t+\tau)f(t)\} = R(\tau) \quad (\text{for any time } t).$$

A stochastic process $f(t)$ is ergodic if its time averages equal its ensemble averages.

We will assume the earth tilt data as an ergodic stationary time series in the design of single-channel predictor.

B. THEORY OF SINGLE-CHANNEL PREDICTOR

1. Method of Approach

a. Inelegant Method

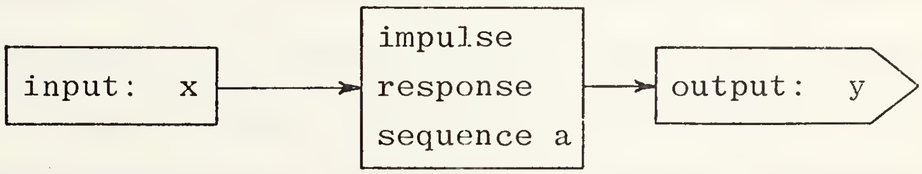
By using the FFT (Fast Fourier Transform) we can determine the spectral density function of a time series. After we invert the spectral density function to the time domain we can get a Fourier series representation of the time series. We then extrapolate to future time to obtain a crude singe-channel prediction.

b. Elegant Method

The Wiener-Hopf equation can be used to design an optimum filter for a single channel predictor. The solution of the Wiener-Hopf equation is facilitated on a high-speed digiter computer by using the Levison recursive method which can be programmed. We will discuss this method of prediction in the following sections.

2. Block Diagram of a Linear Time-Invariant Digital Filter

The following is a block diagram of the input/output relationship of a linear-system:



x is the input signal to the linear-system, y is the output signal from the linear-system. The relation between x and y is:

$$y = x * a$$

where a is the impulse response sequence of the linear-system.

The above equation is the time-domain relation, which says the output y is equal to the convolution of the input x and impulse response sequence a of the system.

In transform domain, the input-output relationship simply becomes:

$$Y(Z) = X(Z)A(Z).$$

That is, the Z-transform of the output signal y is equal to the product of the Z-transform of the input signal x and the Z-transform of the impulse response sequence a of the system. A system that generates its output in this way is called a linear time-invariant digital operator, or a linear time-invariant digital filter. The coefficients of the impulse response sequence are called the operator coefficients or filter coefficients.

3. Least-Squares Shaping Filter

From section 2.2, we know in a linear time-invariant system if we are given the input signal x and the impulse response sequence a of the system, we can get the output signal y by convolving x and a. On the other hand, if we are given the input signal x and we want the output signal y to be some desired form, how should we choose the impulse response sequence or filter coefficients a? Clearly this is a deconvolution problem. In transform-domain it becomes a division problem. That is,

$$A(Z) = \frac{Y(Z)}{X(Z)}$$

where $A(Z)$ is the Z-transform of the desired filter coefficients a. But we know from elementary algebra that even $Y(Z)$ and $X(Z)$ are both of finite terms, $A(Z)$ may not be always of finite terms. In fact in most cases $A(Z)$ will be a polynomial of infinitely many terms. This is to mean we want a filter with infinite-length. If in the real-world situation we can only build a finite-length filter, then we can not obtain the exactly desired output signal y. The difference between the actual output and the desired output is the error. For a specific finite-length filter, the optimum one is that one which will yield an output with smallet error. Like the regression analysis in statistics, here the smallest error means the least squared error. If we denote the desired output signal to be y and the actual output signal to be $\hat{y}$, we want to minimize

$$\sum_{t=0}^{n} (y_t - \hat{y}_t)^2$$

The optimum filter which can do this job is called a least-squares shaping filter since it can generate an output with least squared error to the desired shape of output.

4. Discrete-Time Wiener-Hopf Equation

In this section we will discuss how to determine the least-squares shaping filter's coefficients. Let the input signal be:

$$x = (x_0, x_1, \cdots, x_n)$$

let the desired output signal be

$$y = (y_0, y_1, \cdots, y_{n+m})$$

let the filter coefficients be

$$f = (f_0, f_1, \cdots, f_m)$$

The actual output signal $\hat{y}$ is the convolution of the input signal x and the filter coefficients f, that is:

$$\hat{y} = x * f.$$

Now we want to choose f which will minimize

$$I = \sum_{t=0}^{n+m} (y_t - \hat{y}_t)^2 = \sum_{t=0}^{n+m} (y_t - \sum_{s=0}^{m} f_s x_{t-s})^2.$$

The quantity I is minimized if its partial derivatives with respect to each of the coefficients $f_0, f_1, \cdots, f_m$ equal zero. Therefore we want:

$$\frac{\partial I}{\partial f_j} = \sum_{t=0}^{n+m} 2(y_t - \sum_{s=0}^{m} f_s x_{t-s})(-x_{t-j}) = 0$$

$$(j = 0, 1, 2, \cdots, m)$$

simplifying, we obtain:

$$-\sum_{t=0}^{n+m} y_t x_{t-j} + \sum_{t=0}^{n+m} (\sum_{s=0}^{m} f_s x_{t-s})\, x_{t-j} = 0$$

$$(j = 0, 1, 2, \cdots, m)$$

or

$$\sum_{s=0}^{m} f_s \sum_{t=0}^{n+m} x_{t-s} x_{t-j} = \sum_{t=0}^{n+m} y_t\, x_{t-j}:$$

$$(j = 0, 1, 2, \cdots, m)$$

If we use the notation

$$r_{j-s} = \sum_{t=0}^{n+m} x_{t-s}\, x_{t-j}$$

and

$$g_j = \sum_{t=0}^{n+m} y_t\, x_{t-j}$$

where r_{j-s} is the autocorrelation (for index j-s) of the input signal x and g_j is the cross-correlation (for index j) of the output signal y and the input signal x, we obtain

$$\sum_{s=0}^{m} f_s\, r_{j-s} = g_j \quad (j = 0,\ 1,\ 2,\ \cdots,\ m).$$

After writing out this set of m+1 equations, we have

$$f_0 r_0 + f_1 r_{-1} + f_2 r_{-2} + \cdots + f_m r_{-m} = g_0$$

$$f_0 r_1 + f_1 r_0 + f_2 r_{-1} + \cdots + f_n r_{1-m} = g_1$$

$$f_0 r_2 + f_1 r_1 + f_2 r_0 + \cdots + f_m r_{2-m} = g_2$$

$$\vdots$$

$$f_0 r_m + f_1 r_{m-1} + f_2 r_{m-2} + \cdots + f_m r_0 = g_m.$$

or in matrix form

$$
\begin{bmatrix}
r_0 & r_{-1} & r_{-2} & \cdots & r_m \\
r_1 & r_0 & r_{-1} & \cdots & r_{1-m} \\
r_2 & r_1 & r_0 & \cdots & r_{2-m} \\
 & & \vdots & & \\
r_m & r_{m-1} & r_{m-2} & \cdots & r_0
\end{bmatrix}
\begin{bmatrix}
f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_m
\end{bmatrix}
=
\begin{bmatrix}
g_0 \\ g_1 \\ g_2 \\ \vdots \\ g_m
\end{bmatrix}
$$

The above set of equations is called the normal equations and is the discrete-time Wiener-Hopf equation.

5. <u>A Computer Program for a Single-Channel Predictor</u>

In a least-squares shaping filter system, if the desired output signal y is equal to the input signal x shifted t_o time units ahead, that is

$$y_t = x_{t+t_o}$$

then the filter becomes a predictor since its objective is to predict the value of x_t t_o time units ahead. In this case, the discrete-time Wiener-Hopf equation

$$\sum_{s=0}^{m} f_s \, r_{j-s} = g_j \quad (j = 0, 1, 2, \cdots, m)$$

simply becomes

$$\sum_{s=0}^{m} f_s \, r_{j-s} = r_{j+t_o} \quad (j = 0, 1, 2, \cdots, m)$$

because

$$g_j = \sum_{t=0}^{n+m} y_t \, x_{t-j} = \sum_{t=0}^{n+m} x_{t+t_o} \, x_{t-j} = r_{j+t_o}$$

Writing out the equations in matrix form, and by noting $r_\tau = r_{-\tau}$ for a real-value signal, we obtain

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_m \\ r_1 & r_0 & r_1 & \cdots & r_{m-1} \\ r_2 & r_1 & r_0 & \cdots & r_{m-2} \\ & & \vdots & & \\ r_m & r_{m-1} & r_{m-2} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} = \begin{bmatrix} r_{t_o} \\ r_{t_o+1} \\ r_{t_o+2} \\ \vdots \\ r_{t_o+m} \end{bmatrix}$$

Although we must solve the above m+1 equations to find the filter coefficients $f_0, f_1, \cdots, f_m$, we can take advantage of the special form of the autocorrelation matrix

$$R_m = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_m \\ r_1 & r_0 & r_1 & \cdots & r_{m-1} \\ r_2 & r_1 & r_0 & \cdots & r_{m-2} \\ & & \vdots & & \\ r_m & r_{m-1} & r_{m-2} & \cdots & r_0 \end{bmatrix}$$

in order to reduce the computational work.

The autocorrelation matrix has the property that all the elements on any given diagonal are the same. It is also

a symmetric matrix. Hence the entire autocorrelation matrix R_m of order m+1 does contain only m+1 distinct elements, namely r_0, r_1, r_2, $\cdots$, r_m. Let us now see how we can take advantage of this structure.

The method we use is called the Levinson recursive method. Suppose that we want to solve the following matrix equation:

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_m \\ r_1 & r_0 & r_1 & \cdots & r_{m-1} \\ r_2 & r_1 & r_0 & \cdots & r_{m-2} \\ & & \vdots & & \\ r_m & r_{m-1} & r_{m-2} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} f_0 \\ f_1 \\ f_2 \\ \vdots \\ f_m \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix}$$

where the autocorrelation coefficients r_0, r_1, r_2, $\cdots$, r_m as well as the cross-correlation coefficients g_0, g_1, g_2, $\cdots$, g_m represent the known quantities, and the filter coefficients f_0, f_1, f_2, $\cdots$, f_m represent the unknown quantities. The recursive procedure solves the equation in a step-wise manner. The step k=0 is given as an initial condition, and then the steps k=1, k=2, $\cdots$, k=m are done successively in a recursive manner. The desired filter coefficients are the ones which result on the completion of the final step, namely step k=m.

Let us now show how we proceed from step k=n to step k=n+1. The completion of step k=n requires that we have computed the following quantities:

$$a_{n0},\ a_{n1},\ \cdots,\ a_{nn};\ \alpha_n,\ \ \beta_n$$

and

$$f_{n0},\ f_{n1},\ \cdots,\ f_{nn};\ \gamma_n.$$

By definition, these quantities satisfy the matrix equations:

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \vdots & & \\ r_{n+1} & r_n & r_{n-1} & & r_0 \end{bmatrix} \begin{bmatrix} a_{n0} \\ a_{n1} \\ \vdots \\ a_{nn} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_n \\ 0 \\ \vdots \\ 0 \\ \beta_n \end{bmatrix} \qquad (1)$$

and

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \vdots & & \\ r_{n+1} & r_n & r_{n-1} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} f_{n0} \\ f_{n1} \\ \vdots \\ f_{nn} \\ 0 \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_n \\ \gamma_n \end{bmatrix} \qquad (2)$$

Because of the special structure of the autocorrelation matrix, equation (1) may be written in the following equivalent form:

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \vdots & & \\ r_{n+1} & r_n & r_{n-1} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} 0 \\ a_{nn} \\ \vdots \\ a_{n1} \\ a_{n0} \end{bmatrix} = \begin{bmatrix} \beta_n \\ 0 \\ \vdots \\ 0 \\ \alpha_n \end{bmatrix} \qquad (3)$$

Let C_n be a constant which we will determine in order to get a desired result. The desired result is after we multiply equation (3) by C_n and add the result to equation (1), we want it to be identical to a new equation. That is, we want equation (4) and (5) to be identical.

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \vdots & & \\ r_{n+1} & r_n & r_{n-1} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} a_{n0} + 0 \\ a_{n1} + c_n a_{nn} \\ \vdots \\ a_{nn} + c_n a_{n1} \\ 0 + c_n a_{n0} \end{bmatrix} = \begin{bmatrix} \alpha_n + c_n \beta_n \\ 0 \\ \vdots \\ 0 \\ \beta_n + c_n \alpha_n \end{bmatrix} \qquad (4)$$

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \vdots & & \\ r_{n+1} & r_n & r_{n-1} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} a_{n+1,0} \\ a_{n+1,1} \\ \vdots \\ a_{n+1,n} \\ a_{n+1,n+1} \end{bmatrix} = \begin{bmatrix} \alpha_{n+1} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

In order for equations (4) and (5) to be identical, we require that

$$\begin{bmatrix} \alpha_n + c_n\beta_n \\ 0 \\ \vdots \\ 0 \\ \beta_n + c_n\alpha_n \end{bmatrix} = \begin{bmatrix} \alpha_{n+1} \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

and

$$\begin{bmatrix} a_{n0} + 0 \\ a_{n1} + c_n a_{nn} \\ \vdots \\ a_{nn} + c_n a_{n1} \\ 0 + c_n a_{n0} \end{bmatrix} = \begin{bmatrix} a_{n+1,0} \\ a_{n+1,1} \\ \vdots \\ a_{n+1,n} \\ a_{n+1,n+1} \end{bmatrix} \tag{7}$$

From equation (6), we can determine the value of c_n by

$$\beta_n + c_n \alpha_n = 0$$

or

$$c_n = -\frac{\beta_n}{\alpha_n}$$

After c_n has been determined, we can compute all the following quantities:

$$\alpha_{n+1};\ a_{n+1,0},\ a_{n+1,1},\ a_{n+1,2},\ \cdots,\ a_{n+1,n+1}.$$

Next, let

$$\beta_{n+1} = a_{n+1,0} r_{n+1,0} r_{n+2} + a_{n+1,1} r_{n+1} + \cdots + a_{n+1,n+1} r_1$$

then we get the counterpart of equation (1) of step k=n in step k=n+1 as follows:

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+2} \\ r_1 & r_0 & r_1 & \cdots & r_{n+1} \\ r_2 & r_1 & r_0 & \cdots & r_n \\ & & \cdots & & \\ r_{n+2} & r_{n+1} & r_n & & r_0 \end{bmatrix} \begin{bmatrix} a_{n+1,0} \\ a_{n+1,1} \\ \vdots \\ a_{n+1,n+1} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_{n+1} \\ 0 \\ \vdots \\ 0 \\ \beta_{n+1} \end{bmatrix} \qquad (8)$$

Now, rewrite equation (5) as follows;

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \vdots & & \\ r_{n+1} & r_n & r_{n-1} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} a_{n+1,n+1} \\ a_{n+1,n} \\ \vdots \\ a_{n+1,1} \\ a_{n+1,0} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \alpha_{n+1} \end{bmatrix} \qquad (9)$$

Let us multiply equation (9) by a constant d_n, as yet under-undetermined, and then add the result to equation (2). We obtain

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \cdots & & \\ r_{n+1} & r_n & r_{n-1} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} f_{n0}+d_n a_{n+1,n+1} \\ f_{n1}+d_n a_{n+1,n} \\ \vdots \\ f_{nn}+d_n a_{n+1,1} \\ 0 \quad +d_n a_{n+1,0} \end{bmatrix} = \begin{bmatrix} g_0+0 \\ g_1+0 \\ \vdots \\ g_n+0 \\ \gamma_n+d_n\alpha_{n+1} \end{bmatrix} \qquad (10)$$

We want equation (10) to be identical to the following new equation:

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+1} \\ r_1 & r_0 & r_1 & \cdots & r_n \\ r_2 & r_1 & r_0 & \cdots & r_{n-1} \\ & & \vdots & & \\ r_{n+1} & r_n & r_{n-1} & \cdots & r_0 \end{bmatrix} \begin{bmatrix} f_{n+1,0} \\ f_{n+1,1} \\ \vdots \\ f_{n+1,n} \\ f_{n+1,n+1} \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_n \\ g_{n+1} \end{bmatrix} \qquad (11)$$

This means we require

$$\begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_n \\ \gamma_n + d_n \alpha_{n+1} \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_n \\ g_{n+1} \end{bmatrix} \tag{12}$$

and

$$\begin{bmatrix} f_{n0} + d_n a_{n+1,n+1} \\ f_{n1} + d_n a_{n+1,n} \\ \vdots \\ f_{nn} + d_n a_{n+1,1} \\ d_n a_{n+1,0} \end{bmatrix} = \begin{bmatrix} f_{n+1,0} \\ f_{n+1,1} \\ \vdots \\ f_{n+1,n} \\ f_{n+1,n+1} \end{bmatrix} \tag{13}$$

From equation (12), we can determine d_n by the formula

$$d_n = \frac{g_{n+1} - \gamma_n}{\alpha_{n+1}} .$$

After d_n is substituted in equation (13), we can compute the following quantities:

$$f_{n+1,0}, f_{n+1,1}, f_{n+1,2}, \ldots, f_{n+1,n+1} .$$

Next, let

$$\gamma_{n+1} = f_{n+1,0}r_{n+2}+f_{n+1,1}r_{n+1}+\cdots+f_{n+1,n+1}r_1$$

We obtain the counterpart of equation (2) of step k=n in step k=n+1 as follows:

$$\begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{n+2} \\ r_1 & r_0 & r_1 & \cdots & r_{n+1} \\ r_2 & r_1 & r_0 & \cdots & r_n \\ & & \vdots & & \\ r_{n+2} & r_{n+1} & r_n & \cdots & r_0 \end{bmatrix} \begin{bmatrix} f_{n+1,0} \\ f_{n+1,1} \\ \vdots \\ f_{n+1,n+1} \\ 0 \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ \vdots \\ g_{n+1} \\ \gamma_{n+1} \end{bmatrix} \tag{14}$$

We have completed the recursive procedure from step k=n to step k=n+1. To start at step k=0, we must determine

a_{00}; α_0, β_0

and

f_{00}; γ_0

which satisfy the matrix equations:

$$\begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix} \begin{bmatrix} a_{00} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_0 \\ \beta_0 \end{bmatrix} \tag{15}$$

and

$$\begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix} \begin{bmatrix} f_{00} \\ 0 \end{bmatrix} = \begin{bmatrix} g_0 \\ \gamma_0 \end{bmatrix} \tag{16}$$

In equation (15), we set $a_{00} = 1$ and solve α_0, β_0; we obtain

$$\alpha_0 = r_0, \; \beta_0 = r_1.$$

From equation (16), we obtain

$$f_{00} = \frac{g_0}{r_0}, \quad \gamma_0 = r_1 f_{00}.$$

Starting from step k=0 as above, we can proceed recursively to steps k=1, 2, 3, $\cdots$, m. At the final step k=m, the values obtained for the filter coefficients, namely

$$f_{m0}, f_{m1}, f_{m2}, \cdots, f_{mm}$$

will be the required solution. A computer program based on the above recursive method is included in Appendix A.

III. CHARACTERISTICS OF TIDAL DATA

A. INTRODUCTION

By Newton's law of universal gravitation, each body in the universe attracts another body with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. We may express this fact in a formula:

$$F_g = \frac{Gm_1m_2}{r^2}$$

where F_g is the gravitational force, m_1 and m_2 are the masses of the two bodies in the universe and r is the distance between them. G is called the universal constant of gravitation and is approximately equal to 6.67×10^{-11} $n \cdot m^2/kg^2$.

Celestial bodies such as the earth, the sun, and the moon are, of course, obeying the law of gravitation. Because they are not point masses and have large dimensions, different points on the surface of the body will be affected by different forces. Tidal forces are generated due to this effect. In this chapter we will give a general description of the characteristics of the earth tidal data.

B, GRAVITATIONAL POTENTIAL AND TIDAL POTENTIAL

1. Basic Formulas From Mathematics

Since in the derivation of tidal potential we will use some formulas from applied mathematics and spherical

trigonometry, we give a brief discussion of them in this section.

a. Legendre Polynomials

The Legendre Polynomials are defined by:

$$P_n(x) = \frac{(2n-1)(2n-3)\ldots1}{n!}\left\{x^n - \frac{n(n-1)}{2(2n-1)}x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2.4(2n-1)(2n-3)}x^{n-4} - \ldots\right\}.$$

We observe that $P_n(x)$ is a polynomial of degree n. The first few Legendre polynomials are as follows:

$$P_0(x) = 1 \qquad P_3(x) = 1/2(5x^3 - 3x)$$

$$P_1(x) = x \qquad P_4(x) = 1/8(35x^4 - 30x^2 + 3)$$

$$P_2(x) = 1/2(3x^2 - 1) \qquad P_5(x) = 1/8(63x^5 - 70x^3 + 15x).$$

The Legendre polynomials can also be expressed by Rodrique's formula:

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n.$$

After we have obtained $P_0(x)$ and $P_1(x)$ by the formulas given above, we can also use the following recurrence formula to obtain the higher degree Legendre Polynomials:

$$P_{n+1}(x) = \frac{2n+1}{n+1}xP_n(x) - \frac{n}{n+1}P_{n-1}(x).$$

The function $\dfrac{1}{\sqrt{1-2xt+t^2}}$ is called the generating function for Legendre polynomials. By using the binomial theorem we can prove the following result:

$$\frac{1}{\sqrt{1-2xt+t^2}} = \sum_{n=0}^{\infty} P_n(x)t^n \, .$$

b. The Cosine Law for Spherical Triangles

In a spherical triangle, if we denote the three sides to be a, b, c and the three angles to be A, B, C; the following relations hold.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$

We call the above formulas the cosine law for spherical triangles.

2. Derivation of Gravitational Potential and Tidal Potential.

a. Gravitational Force, Gravitational Field Intensity, and Gravitational Potential

According to Newton's law of universal gravitation, a point mass m_0 at a distance r from another body of mass M will experience a force

$$\vec{F}_g = -G \frac{m_0 M}{r^2} \vec{i}_r$$

where $\vec{i}_r$ is a unit vector in the direction of r. $\vec{F}_g$ is called the gravitational force. The minus sign denotes it is an attractive force.

We define the gravitational field intensity at a point to be the gravitational force experienced by a unit mass at that point. If the unit mass is at a distance r

from the body with mass M, we will have

$$\vec{I}_g = -G \frac{M}{r^2} \vec{i}_r$$

where $\vec{I}_g$ denotes the gravitational field intensity.

The gravitational potential V_g at a point which lies at a distance r from a body with mass M is defined as follows:

$$V_g = \int_r^{\infty} \vec{I}_g \cdot d\vec{r} = \int_r^{\infty} -G \frac{M}{r^2} \vec{i}_r \cdot d\vec{r}$$

$$= \int_r^{\infty} -G \frac{M}{r^2} dr = -G \frac{M}{r}.$$

Another way to show the relation between V_g and $\vec{I}_g$ is

$$\vec{I}_g = -\nabla V_g = -\text{grad } V_g$$

Since V_g is a scalar while $\vec{F}_g$ and $\vec{I}_g$ are vectors, V_g is much easier to handle than $\vec{F}_g$ and $\vec{I}_g$. In deriving the theoretical tide on earth induced by the moon and the sun, we will use tidal potential instead of tidal force.

b. Gravitational Potential and tidal potential at a Point on the Surface of Earth

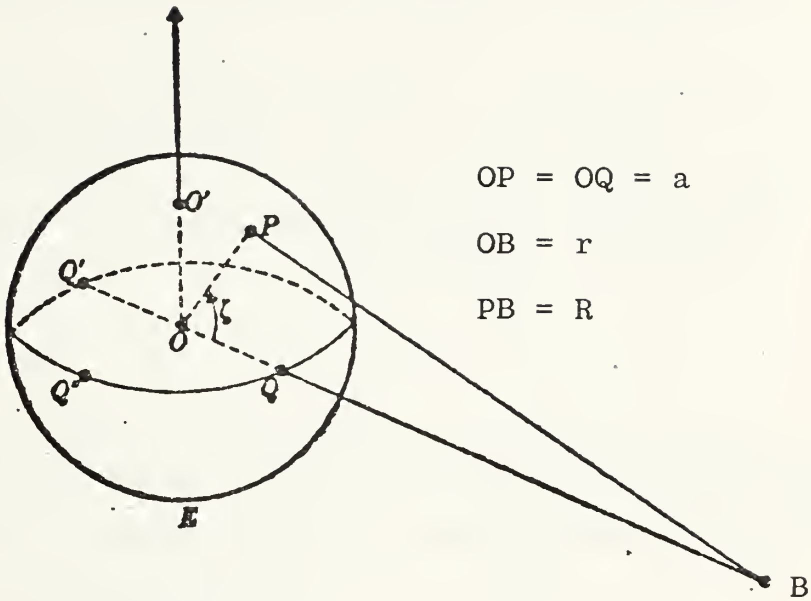


Figure 1

In Figure 1, let E be the earth, P is any point on the surface of E. The gravitational potential at P created by a celestial body of mass M is

$$V = -G\,\frac{M}{R}$$

where R is the distance PB. Notice we are assuming that the whole mass of the celestial body is concentrated at a point B. By the cosine law for plane triangles, we have:

$$R^2 = a^2 + r^2 - 2ar\cos\xi.$$

Therefore we can rewrite the gravitational potential V as follows:

$$V = -G\,\frac{M}{\sqrt{a^2 + r^2 - 2ar\cos\xi}}$$

$$= -G\,\frac{M}{r}\left[1-2\left(\frac{a}{r}\right)\cos\xi + \left(\frac{a}{r}\right)^2\right]^{-1/2}$$

After we expand the above equation, we get a more compact form:

$$v = -G \frac{M}{r} \sum_{n=0}^{\infty} P_n(\cos \xi)\left(\frac{a}{r}\right)^n$$

where $P_n(\cos \xi)$ are the Legendre polynomials. The first term of V is

$$V_0 = -G \frac{M}{r},$$

which is a constant and has no contribution to the gravitational force. Therefore it has no physical significance and we will disregard it. The second term is

$$V_1 = -G \frac{M}{r} (\cos \xi) \frac{a}{r} = -G \frac{M}{r^2} \ a \cos \xi .$$

To evaluate the contribution of V_1 to the gravitational force, let us choose a cartesian frame of reference centered at O, the x axis running in the direction OB and the y axis running in the direction OO'. With this choice of coordinate system we have

$$a \cos \xi = x \quad , \qquad a \sin \xi = y \quad ,$$

and the contribution of V_1 to the gravitational field intensity is

$$\vec{I_1} = -\nabla V_1 = -\nabla(-G \frac{M}{r^2} x) = G \frac{M}{r^2} \ \vec{i_x}$$

where $\vec{i_x}$ is a unit vector in the direction of x axis. This shows that $\vec{I_1}$ is a uniform gravitational field. It has a constant magnitude $G \frac{M}{r^2}$ and a constant direction which is parallel to the x axis. Hence V_1 will not create tidal effect.

The third term of V is

$$V_2 = -G\,\frac{M}{r} \cdot 1/2\,(3\cos^2\xi - 1)\,\left(\frac{a}{r}\right)^2$$

$$= -G\,\frac{M}{r^3} \cdot 1/2\,(3a^2\cos^2\xi - a^2)$$

$$= -G\,\frac{M}{r^3} \cdot 1/2\,(3x^2 - x^2 - y^2)$$

$$= -G\,\frac{M}{r^3} \cdot 1/2\,(2x^2 - y^2)$$

in our choice of coordinate system. The gravitational field intensity created by V_2 is

$$\vec{I}_2 = -\nabla V_2 = -\nabla\left[-G\,\frac{M}{2r^3}\,(2x^2 - y^2)\right]$$

$$= G\,\frac{M}{r^3}\,(2\,x\,\vec{i}_x - y\,\vec{i}_y)$$

where $\vec{i}_x$ and $\vec{i}_y$ are unit vectors in the direction of x axis and y axis. We observe that $\vec{I}_2$ is different at different locations because it depends upon x and y. This non-uniform gravitational field intensity will create a tidal force at all points on the earth. Hence V_2 is responsible for the tidal effect. We can also show that after V_2, every term of V is responsible for the tidal effect. Therefore we call

$$V_T = -G\,\frac{M}{r}\sum_{n=2}^{\infty} P_n(\cos\xi)\,\left(\frac{a}{r}\right)^n$$

the tidal potential.

3. Tidal Potential on the Surface of Earth Induced by the Moon and the Sun

Although theoretically every celestial body in the universe creates a tidal potential on our earth, they are so

small that in practice we can neglect all of them except those created by the moon and the sun. If we denote the tidal potential at a point P on the earth induced by the moon be V_m, by the sun be V_s, we have

$$V_m = -G \frac{M_m}{r_m} \sum_{n=2}^{\infty} P_n(\cos \xi_m) \left(\frac{a}{r_m}\right)^n$$

$$V_s = -G \frac{M_s}{r_s} \sum_{n=2}^{\infty} P_n(\cos \xi_s) \left(\frac{a}{r_s}\right)^n$$

where

M_m = the mass of the moon $\doteq 7.38 \times 10^{22}$ kg,

M_s = the mass of the sun $\doteq 1.991 \times 10^{30}$ kg,

r_m = the distance from the center of the earth to the center of the moon,

r_s = the distance from the center of the earth to the center of the sun,

ξ_m = the zenith angle of the moon,

ξ_s = the zenith angle of the sun, and

a = the radius of the earth $\doteq$ 6365 km.

If we use the mean value of r_m and the mean value of r_s to calculate $\frac{a}{r_m}$ and $\frac{a}{r_s}$, we obtain

$$\frac{a}{r_m} \doteq \frac{6365}{3 \cdot 84 \times 10^5} \doteq 1 \cdot 66 \times 10^{-2}$$

$$\frac{a}{r_s} \doteq \frac{6365}{1 \cdot 49 \times 10^8} \doteq 4 \cdot 33 \times 10^{-5}$$

Since $\frac{a}{r_m}$ and $\frac{a}{r_s}$ are both very small, we can express V_m and V_s approximately as follows:

$$V_m \doteq -G \frac{M_m}{r_m} P_2 (\cos \xi_m) \left(\frac{a}{r_m}\right)^2$$

$$= -G \frac{M_m a^2}{r_m^3} \cdot 1/2 \, (3\cos^2 \xi_m - 1),$$

$$V_s \doteq -G \frac{M_s}{r_s} P_2 (\cos \xi_s) \left(\frac{a}{r_s}\right)^2$$

$$= -G \frac{M_s a^2}{r_s^3} \cdot 1/2 \, (3\cos^2 \xi_s - 1).$$

For the same value of ξ_m and ξ_s, the ratio is

$$\frac{V_m}{V_s} \doteq \frac{M_m}{M_s} \left(\frac{r_s}{r_m} \right)^3 \doteq 2 \cdot 17$$

This indicates that the tidal potential due to the moon is roughly twice that due to the sun. Therefore the moon should be the preponderant source of the tidal effect on our earth, as is indeed observed.

C. THE PERIODICITIES IN THE TIDAL POTENTIAL INDUCED BY THE MOON AND THE SUN

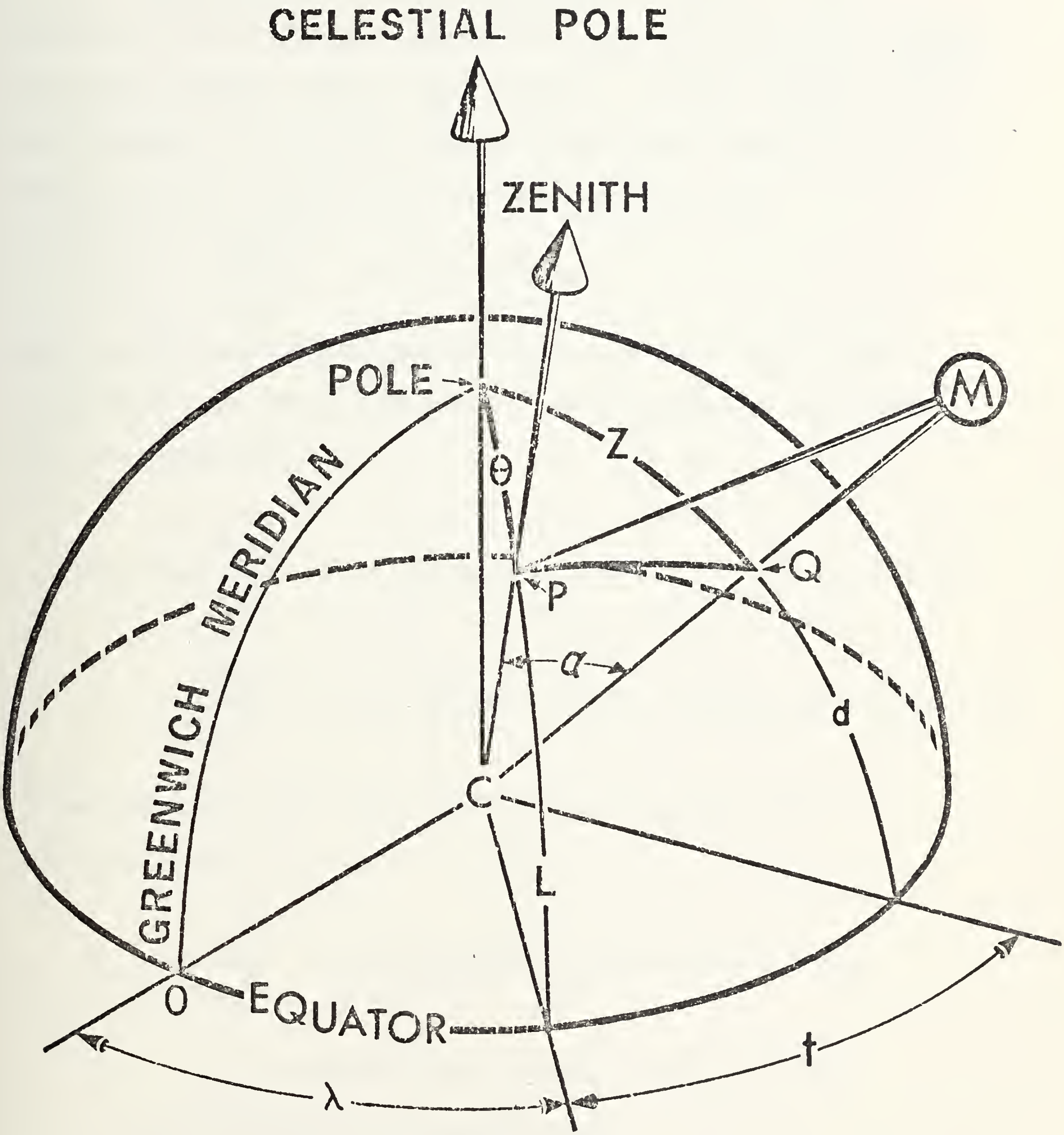


Figure 2

In Figure 2, let P be a point on the earth with latitude L and longitude λ, Q be the geographical point of the celestial body M which has declination d and local hour angle t at the instant shown in the figure. If the celestial body is the moon, then by the result of the last section we can write the tidal potential at P induced by it as:

$$V_m \doteq -G \frac{M_m a^2}{r_m^3} \cdot 1/2\ (3 \cos^2 \alpha - 1).$$

Now we will derive the periodicities present in the tidal potential. By using the cosine law for spherical triangles, we can rewrite V_m as

$$V_m \doteq -G \frac{M_m a^2}{r_m^3} \cdot 1/2 \left[3(\cos \theta \cos Z + \sin \theta \sin Z \cos t)^2 - 1\right]$$

$$= -G \frac{M_m a^2}{r_m^3} \cdot 1/2 \left[3(\sin L \sin d + \cos L \cos d \cos t^2 - 1\right]$$

After we expand the above equation and algebraically simplify, we obtain

$$V_m \doteq -G \frac{M_m a^2}{r_m^3} \cdot 3/4 \left[3(\sin^2 L - 1/3)\ (\sin^2 d - 1/3)\right.$$

$$\left. + \sin 2L \sin 2d \cos t + \cos^2 L \cos^2 d \cos 2t\right]$$

$$= K_m \left[3(\sin^2 L - 1/3)\ (\sin^2 d - 1/3\right]$$

$$+ k_m(\sin 2L \sin 2d \cos t) \quad + K_m(\cos^2 L \cos^2 d \cos 2t)$$

where

$$K_m = -3/4\ G\ \frac{M_m a^2}{r_m^3}$$ is a constant.

L is the latitude of P and is also a constant. d and t are the declination and local hour angle of the moon, respectively. They are changing from time to time. t will go through a complete cycle of 360^o in one lunar day which is about 24 hours and 50 minutes on the average. d will go through a complete cycle in about one month. Since in one lunar day d will change very little, we will consider the term $K(\sin 2L \sin 2d \cos t)$ as the tidal potential component with frequency 1 cycle/lunar day and the term $K(\cos^2 L \cos^2 d \cos 2t)$ with frequency 2 cycle/lunar day. We also call them diurnal and semi-diurnal component. The first term $K\left[3(\sin^2 L-1/3)(\sin^2 d-1/3)\right]$ contains $\sin^2 d$ which is an even function of d. Since d goes from zero to positive maximum and then back to zero and then goes to negative in a symmetric way, $K\left[3(\sin^2 L-1/3)(\sin^2 d-1/3)\right]$ will be a component with frequency 1 cycle per half month. We call it the fortnightly component.

By the same procedure we can express the tidal potential at the point P induced by the sun as

$$V_s = K_s\left[3(\sin^2 L-1/3)(\sin^2 d-1/3)\right]$$
$$+ K_s(\sin 2L \sin 2d \cos t) + K_s(\cos^2 L \cos^2 d \cos 2t)$$

where $K_s = -3/4\ G\ \frac{M_s a^2}{r_s^3}$.

d and t of the sun go through a complete cycle with periods one year and one solar day, respectively. Hence the three terms of V_s have frequencies 1 cycle/half year, 1 cycle/solar day, and 2 cycle/solar day.

If we do not neglect the higher order terms in the tidal potential, and also consider the quite complicated relative motion of the moon and the sun with respect to our earth in detail, the spectrum of the tidal potential should be composed of an infinite number of terms. But for practical purposes it is quite enough to consider only the most predominant nine terms. Table 1 is a summary of these terms.

NAME	PERIOD (hr)	FREQUENCY (degree/hr)	FREQUENCY (rad./hr)
K1	0.23934464E 02	0.15041068E 02	0.26251602E 00
O1	0.25819336E 02	0.13943035E 02	0.24335176E 00
P1	0.24065887E 02	0.14958931E 02	0.26108247E 00
Q1	0.26868347E 02	0.13398661E 02	0.23385066E 00
M2	0.12420603E 02	0.28984100E 02	0.50586778E 00
S2	0.12000000E 02	0.30000000E 02	0.52359855E 00
N2	0.12658348E 02	0.28439728E 02	0.49636668E 00
K2	0.11967240E 02	0.30082123E 02	0.52503186E 00
M4	0.62103014E 01	0.57968201E 02	0.10117350E 01

TABLE 1. Summary of Nine Principal Tidal Lines.

D. MISCELLANEOUS NOISE IN THE REAL TIDAL DATA

Meteorological activities such as the rainfall, diurnal and seasonal temperature variations and pressure changes, introduce noise in the tidal tilt data. Other factors such as the ocean loading effect, daily and weekly traffic patterns also introduce some noise. However, to a lesser degree, these effects can be considered stationary and predictable over short periods of time. The effects due to local earthquake activity or underground nuclear tests would introduce excessive noise into the real tidal tilt data. These noises are random in nature compared to other noise. We will remove these noises from the real tidal data when we use them as the input data of the single-channel predictor.

IV. APPLICATIONS OF SINGLE-CHANNEL PREDICTOR

A. DISCUSSION OF THE PARAMETER'S VALUE USED IN THE COMPUTER PROGRAM

Before using the "PREDICT" computer program, the first step is to convert a stationary continuous time signal to a discrete one by the method of sampling. The sampling period T should be chosen according to the following formula:

$$T < \frac{1}{2f_m}$$

where f_m is the highest frequency component of the time signal. After we have obtained the discrete time signal, the next problem we will consider is how to choose the length of the input data; how far ahead should we predict and how long the filter length should be. Normally we should first do a spectral analyssi of the signal; find out the highest and the lowest frequency components. Then we should choose the length of the input signal data greater than the longest period of the spectral components. For the predicted distance, it is a good rule to choose it less than one-twelfth of the longest period of the spectral components and at the same time close to a small multiple of the shortest period of the spectral components. For the earth tidal tilt data, we have shown in chapter III that the most important spectral components are semi-diurnal, diurnal and fortnightly. Hence we choose the sampling period as one hour which is

considerably less than $1/2f_m$ (about 6 hours). Also, we choose the length of the input data to be 720 hours which is a little more than twice the period of the fortnightly components. The predicted distance we use is 24 hours which is less than one-twelfth of a fortnight and at the same time almost equal to two periods of the semi-diurnal components and one period of the diurnal components.

Another parameter used in the "PREDICT" program is the length of the filter coefficients. It is the most important parameter which affects the computer CPU time used to do all the computations of the program. We will discuss it in detail in the next section.

B. AN INVESTIGATION OF THE FILTER LENGTH USED IN THE "PREDICT" PROGRAM

For a signal of pure sine wave form, it is easy to show that a filter length of two is enough to predict the future data. From the basic trigonometry, we have

$$\sin (t+T) = \sin t \cos T + \cos t \sin T \tag{1}$$

$$\sin (t-T) = \sin t \cos T - \cos t \sin T \tag{2}$$

By adding equation (1) and (2) together, we obtain

$$\sin (t+T) + \sin (t-T) = 2 \cos T \sin t \tag{3}$$

or

$$\sin (t+T) = 2 \cos T \sin t + (-1) \sin (t-T) \tag{4}$$

Equation (4) means we can use a two-length filter coefficients

$$(2 \cos T, -1)$$

to predict one unit sampling period T ahead if the signal is a pure sine wave. More specifically, let us suppose there is a sine wave signal with maximum amplitude 1 and sampling at a uniform time interval T which corresponds to 30 degrees. The discrete time signal will be

$$(\ldots, \underset{\uparrow}{0}, \frac{1}{2}, \frac{\sqrt{3}}{2}, 1, \frac{\sqrt{3}}{2}, \frac{1}{2}, 0, -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -1,$$
$$-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0, \ldots).$$

The two-length filter coefficients in this case are,

$$(2 \cos 30^{o}, -1) = (\sqrt{3}, -1).$$

If we convolve the signal with the filter coefficients, we obtain a new signal

$$(\ldots, 0, \underset{\uparrow}{\frac{1}{2}}, \frac{\sqrt{3}}{2}, 1, \frac{\sqrt{3}}{2}, \frac{1}{2}, 0, -\frac{1}{2}, -\frac{\sqrt{3}}{2}, -1,$$
$$-\frac{\sqrt{3}}{2}, -\frac{1}{2}, 0, \frac{1}{2}, \ldots)$$

which is just the original signal shifted one sampling period ahead. If we want to predict two sampling period ahead, we can obtain the required filter coefficients in a recursive way as follows:

$$\sin(t+2T) = 2\cos T \sin(t+T) + (-1)\sin t$$

$$= 2\cos T\left[2\cos T \sin t + (-1)\sin(t-T)\right] + (-1)\sin t$$

$$= (4\cos^2 T-1)\sin t + (-2\cos T)\sin(t-T)$$

which in our example will be,

$$(4\cos^2 30^{\circ}-1,\ -2\cos 30^{\circ}) = (2,\ -\sqrt{3}).$$

If we convolve the original signal with $(2, -\sqrt{3})$, we will obtain a signal,

$$(\ldots,\ \underset{\uparrow}{\frac{\sqrt{3}}{2}},\ 1,\ \frac{\sqrt{3}}{2},\ \frac{1}{2},\ 0,\ -\frac{1}{2}\ -\frac{\sqrt{3}}{2},\ -1,\ -\frac{\sqrt{3}}{2}\ -\frac{1}{2},\ 0,\ \frac{1}{2},\ \frac{\sqrt{3}}{2},\ \ldots)$$

which is the original signal shifted 2 sampling periods ahead.

When a signal's spectrum contains more than one frequency, the sampling period T will correspond to different angle measurements for different frequency components. If we want to predict such signals one sampling period T ahead, the filter length in general will not be of finite length. With a finite length filter we can only approximately predict the time shifted signal.

The earth tidal tilt data contains nine principal spectral components and various sources of noises. To predict the earth tidal tilt data by using the "PREDICT" program, the best way to choose the filter length is by doing a statistical analysis for some samples and then make a decision. Appendix B contains a summary of test results of RMS error

VS. filter length used in the "PREDICT" program. Seven sample runs have been made. The conclusions are as follows:

(1) The RMS error of the last 24 hours is much smaller than the RMS error of the whole 720 hours.

(2) The filter length at 720, which is the maximum length for a 720 input data, causes larger prediction error than other filter lengths.

(3) The filter length between 60 and 660 does not have too much effect on the predicted error, with only a little improvement in the central region.

Based on the above results, we think that normally a filter length of 120 is enough for the prediction. But if we are quite concerned about the accuracy of the prediction, we may use a filter length near 360.

If the length of input data is not 720, the rule of choosing filter length will be 0.2 to 0.5 of the input data length. The length of the input data should be greater than the period of the lowest frequency component of the signal.

C. USING THE SINGLE-CHANNEL PREDICTOR AS AN INTERPOLATOR

If for some reason there is a missing data segment in an otherwise continuous record, we can apply the "PREDICT" program to predict the lost data. Since theoretically the program can be used to predict in both directions, it is a good rule to predict the lost data from both the forward direction and the backward direction and then take the average

value as the predicted data value. It is easy to show that the expected mean squared error of the average value is less than or at most equal to the expected mean squared error caused by one direction prediction. Suppose at an arbitrary point in time the prediction error from the forward direction is E_1, from the backward direction is E_2, the averaged error is $(E_1 + E_2)/2$, we have,

$$\frac{1}{2} E_1^2 + \frac{1}{2} E_2^2 - \left(\frac{E_1+E_2}{2}\right)^2 = \frac{E_1^2 + E_2^2 - 2E_1E_2}{4}$$

$$= \frac{1}{4} (E_1 - E_2)^2 \geq 0.$$

or

$$\frac{1}{2} E_1^2 + \frac{1}{2} E_2^2 \geq \left(\frac{E_1 + E_2}{2}\right)^2$$

The above inequality means whenever we apply the single-channel predictor as an interpolator, we should always predict from both directions and use the averaged prediction value. Appendix C contains the results of six test runs on the real tidal tilt data to illustrate the application of singe-channel predictor.

APPENDIX A

COMPUTER PROGRAM 'PREDICT'

```
C
C     ...................................................................
C
C     PROGRAM 'PREDICT'.
C
C     PURPOSE
C        USED FOR STATIONARY STOCHASTIC TIME SERIES PREDICTION AND
C        INTERPOLATION.
C
C     DESCRIPTION OF PARAMETERS
C        WW      LENGTH OF INPUT DATA TO THE FILTER.
C        WT      PREDICTED DISTANCE.
C        FL      LENGTH OF FILTER COEFFICIENTS.
C        ND      NUMBER OF DATA USED IN THE PROGRAM.
C                ND SHOULD BE EQUAL TO WW+WT.
C        FF      THE ARRAY OF INPUT DATA TO THE PROGRAM.
C        GG      THE TEMPORARY ARRAY OF INPUT DATA TO THE PROGRAM WHEN
C                MAKING BACKWARD PREDICTION.
C        F       THE ARRAY OF INPUT DATA TO THE FILTER.
C        G       THE ARRAY OF DESIRED OUTPUT DATA FROM THE FILTER WHEN
C                DETERMINING THE FILTER COEFFICIENTS. IT IS ALSO THE
C                ARRAY OF INPUT DATA TO THE FILTER WHEN MAKING PREDICTION
C                OR INTERPOLATION.
C        R       THE ARRAY OF AUTOCORRELATION COEFFICIENTS OF F.
C        B       THE ARRAY OF CROSSCORRELATION COEFFICIENTS OF F AND G.
C        FN      THE ARRAY OF OPTIMUM FILTER COEFFICIENTS.
C        T       THE ARRAY OF OUTPUT DATA FROM THE FILTER WHEN MAKING
C                PREDICTION OR INTERPOLATION.
C        P       THE ARRAY OF PREDICTED DATA.
C        ICODE   PREDICTION DIRECTION CODE
C                ICODE=0   FORWARD PREDICTION
C                ICODE=1   BACKWARD PREDICTION.
C
C     METHOD
C        BY MEANS OF N. LEVINSON'S RECURSIVE METHOD TO SOLVE THE
C        DISCRETE-TIME WIENER-HOPF EQUATION.
C
C     PROGRAMMED BY M. D. WOOD, N. E. KING, C. W. CHANG.
C
C     LATEST REVISION    NOV. 1974.
C
C     ...................................................................
C
C     THE FOLLOWING DIMENSIONS AND NUMERICAL VALUES FOR THE PARAMETERS
C     ARE USED IN EARTH TIDAL TILT DATA PREDICTION, FOR OTHER STATIONARY
C     TIME SERIES, SEE SUGGESTIONS IN THIS THESIS.
C
      INTEGER FL,WW,WT
```

```
      DIMENSION FF(744),GG(744),F(720),G(720),R(720),B(720),FN(720),
     CT(720),P(24)
      WW=720
      WT=24
      ND=744
      FL=360
      ICODE=0
C
C     READ IN INPUT DATA.
C
      IF(ICODE.EQ.1) GO TO 3
      READ(5,1) (FF(I),I=1,ND)
      GO TO 4
    3 READ(5,1) (GG(I),I=1,ND)
      DO 5 I=1,ND
    5 FF(I)=GG(ND+1-I)
    4 DO 6 I=1,WW
      F(I)=FF(I)
    6 G(I)=FF(I+WT)
C
C     COMPUTE AUTOCORRELATION COEFFICIENTS OF F AND STORE THE RESULTS
C     IN ARRAY R.
C
      CALL COR(F,F,R,WW,FL)
C
C     COMPUTE CROSSCORRELATION COEFFICIENTS OF F AND G AND STORE THE
C     RESULTS IN ARRAY B.
C
      CALL COR(F,G,B,WW,FL)
C
C     COMPUTE OPTIMUM FILTER COEFFICIENTS BY MEANS OF N. LEVINSON'S
C     RECURSIVE METHOD AND STORE THE RESULTS IN ARRAY FN.
C
      M=FL-1
      CALL FILTER(M,B,R,FN)
C
C     COMPUTE PREDICTED DATA BY CONVOLVING G WITH FN AND STORE THE
C     RESULTS IN ARRAY T.
C
      CALL CONV(G,FN,T,WT,WW,FL)
C
C     WRITE OUT THE LAST 24 DATA OF T WHICH IS THE DATA WE WANT TO
C     PREDICT.
C
      IF(ICODE.EQ.1) GO TO 7
      DO 12 I=1,24
   12 P(I)=T(WW-WT+I)
      GO TO 8
```

```
  7 DO 13 I=1,24                                                    CHA00970
 13 P(I)=T(WW+1-I)                                                  CHA00980
  8 WRITE(6,9)                                                      CHA00990
    WRITE(6,10) (P(I),I=1,24)                                       CHA01000
    WRITE(7,11) (P(I),I=1,24)                                       CHA01010
  1 FORMAT(19X,12F5.0)                                              CHA01020
  9 FORMAT(1H1,49X,'THE PREDICTED DATA ARE AS FOLLOWS')             CHA01030
 10 FORMAT(1H0,5X,12F10.2)                                          CHA01040
 11 FORMAT(12F6.2)                                                  CHA01050
    STOP                                                            CHA01060
    END                                                             CHA01070
    SUBROUTINE COR(F,G,T,WW,FL)                                     CHA01080
    DIMENSION F(720),G(720),T(720)                                  CHA01090
    INTEGER WW,FL                                                   CHA01100
    DO 300 I=1,FL                                                   CHA01110
    SM=0.                                                           CHA01120
    K=I                                                             CHA01130
    L=WW-I+1                                                        CHA01140
    DO 301 J=1,L                                                    CHA01150
    SM=F(J)*G(K)+SM                                                 CHA01160
301 K=K+1                                                           CHA01170
300 T(I)=SM                                                         CHA01180
    RETURN                                                          CHA01190
    END                                                             CHA01200
    SUBROUTINE FILTER (M,B,R,FN)                                    CHA01210
    DIMENSION AN1(720),ANZ(720),B(720),R(720),FN(720)               CHA01220
    ANZ(1)=1.0                                                      CHA01230
    ALFA=R(1)                                                       CHA01240
    BETA=R(2)                                                       CHA01250
    FN(1)=B(1)/R(1)                                                 CHA01260
    GAMMA=FN(1)*R(2)                                                CHA01270
    N=0                                                             CHA01280
100 N=N+1                                                           CHA01290
    N1=N+1                                                          CHA01300
    CKN=-(BETA)/ALFA                                                CHA01310
    ALFA=ALFA+CKN*BETA                                              CHA01320
    IF(ALFA.NE.0.) GO TO 101                                        CHA01330
    RETURN                                                          CHA01340
101 CONTINUE                                                        CHA01350
    BETA=0.                                                         CHA01360
    DO 10 J=1,N1                                                    CHA01370
    IF(J.EQ.1) GO TO 11                                             CHA01380
    IF(J.EQ.N+1) GO TO 12                                           CHA01390
    AN1(J)=ANZ(J)+CKN*ANZ(N-J+2)                                    CHA01400
    IF(N.EQ.M) GO TO 10                                             CHA01410
    BETA=BETA+AN1(J)*R(N-J+3)                                       CHA01420
    GO TO 10                                                        CHA01430
 11 AN1(J)=ANZ(1)                                                   CHA01440
```

```
      IF(N.EQ.M) GO TO 10
      BETA=BETA+AN1(J)*R(N-J+3)
      GO TO 10
   12 AN1(J)=CKN*ANZ(1)
      IF(N.EQ.M) GO TO 10
      BETA=BETA+AN1(J)*R(N-J+3)
   10 CONTINUE
      DO 20 J=1,N1
   20 ANZ(J)=AN1(J)
      QN=(B(N+1)-GAMMA)/ALFA
      GAMMA=0.
      DO 30 J=1,N1
      IF(J.EQ.N+1) GO TO 31
      FN(J)=FN(J)+QN*ANZ(N-J+2)
      IF(N.EQ.M) GO TO 30
      GAMMA=GAMMA+FN(J)*R(N-J+3)
      GO TO 30
   31 FN(J)=QN*ANZ(1)
      IF(N.EQ.M) GO TO 30
      GAMMA=GAMMA+FN(J)*R(N-J+3)
   30 CONTINUE
      IF(N.LT.M) GO TO 100
      RETURN
      END
      SUBROUTINE CONV(G,FN,T,WT,WW,FL)
      DIMENSION G(720),FN(720),T(720)
      INTEGER WT,WW,FL
      T(1)=FN(1)*G(1)
      DO 100 K=2,FL
      TS=0
      DO 110 I=1,K
      TS=FN(I)*G(K+1-I)+TS
  110 CONTINUE
      T(K)=TS
  100 CONTINUE
      IF(FL.EQ.WW) GO TO 140
      KFL=FL+1
      DO 120 K=KFL,WW
      TS=0
      DO 130 I=1,FL
      TS=FN(I)*G(K+1-I)+TS
  130 CONTINUE
      T(K)=TS
  120 CONTINUE
  140 RETURN
      END
```

APPENDIX B

RESULTS OF SEVEN TEST RUNS OF PREDICTION UNDER DIFFERENT FILTER LENGTHS

DATA USED

1. Instrument Number 07

2. Instrument Location

Berkeley	California
City	State

Longitude: $122^{\circ}14'.1$ W

Latitude: $37^{\circ}52'.6$ N

3. Date of Data

Year 1970

Year 1971

TEST RUN NO. 1

FILTER LENGTH	RMS ERROR MONTH	RMS ERROR LAST DAY
60	0.405	0.139
120	0.395	0.132
180	0.386	0.106
240	0.380	0.092
300	0.379	0.089
360	0.370	0.097
420	0.367	0.114
480	0.367	0.117
540	0.365	0.121
600	0.357	0.151
660	0.354	0.143
720	0.349	0.159

TEST RUN NO. 2

FILTER LENGTH	RMS ERROR MONTH	RMS ERROR LAST DAY
60	0.289	0.229
120	0.281	0.238
180	0.276	0.255
240	0.274	0.258
300	0.273	0.257
360	0.273	0.264
420	0.272	0.261
480	0.271	0.272
540	0.270	0.275
600	0.270	0.283
660	0.270	0.292
720	0.270	0.302

TEST RUN NO. 3

FILTER LENGTH	RMS ERROR MONTH	RMS ERROR LAST DAY
60	0.304	0.144
120	0.301	0.135
180	0.299	0.135
240	0.290	0.109
300	0.284	0.131
360	0.282	0.142
420	0.279	0.133
480	0.278	0.127
540	0.278	0.128
600	0.277	0.130
660	0.277	0.131
720	0.277	0.134

TEST RUN NO. 4

FILTER LENGTH	RMS ERROR MONTH	LAST DAY
60	0.393	0.465
120	0.407	0.474
180	0.412	0.472
240	0.413	0.449
300	0.413	0.458
360	0.414	0.463
420	0.417	0.464
480	0.422	0.454
540	0.421	0.464
600	0.419	0.456
660	0.422	0.444
720	0.428	0.615

TEST RUN NO. 5

FILTER LENGTH	RMS ERROR MONTH	RMS ERROR LAST DAY
60	0.194	0.118
120	0.191	0.113
180	0.197	0.117
240	0.200	0.118
300	0.203	0.119
360	0.203	0.119
420	0.205	0.122
480	0.203	0.120
540	0.203	0.121
600	0.204	0.122
660	0.205	0.120
720	0.207	0.144

TEST RUN NO. 6

FILTER LENGTH	RMS ERROR MONTH	LAST DAY
60	0.409	0.129
120	0.414	0.136
180	0.411	0.139
240	0.410	0.134
300	0.410	0.126
360	0.410	0.142
420	0.407	0.135
480	0.408	0.131
540	0.378	0.175
600	0.375	0.190
660	0.376	0.186
720	0.375	0.271

TEST RUN NO. 7

FILTER LENGTH	RMS ERROR MONTH	RMS ERROR LAST DAY
60	0.368	0.174
120	0.357	0.171
180	0.355	0.153
240	0.359	0.168
300	0.350	0.178
360	0.359	0.171
420	0.367	0.165
480	0.367	0.164
540	0.369	0.177
600	0.369	0.194
660	0.371	0.191
720	0.378	0.425

SUMMARY OF RESULTS

RMS ERROR OF WHOLE MONTH UNDER DIFFERENT FILTER LENGTH

FILTER			RMS ERROR OF WHOLE MONTH					
LENGTH	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	MEAN
60	0.405	0.289	0.304	0.393	0.194	0.409	0.368	0.337
120	0.395	0.281	0.301	0.407	0.191	0.414	0.357	0.335
180	0.386	0.276	0.299	0.412	0.197	0.411	0.355	0.334
240	0.380	0.274	0.290	0.413	0.200	0.410	0.359	0.332
300	0.379	0.273	0.284	0.413	0.203	0.410	0.350	0.330
360	0.370	0.273	0.282	0.414	0.203	0.410	0.359	0.330
420	0.367	0.272	0.279	0.417	0.205	0.407	0.367	0.331
480	0.367	0.271	0.278	0.422	0.203	0.408	0.367	0.331
540	0.365	0.270	0.278	0.421	0.203	0.378	0.369	0.326
600	0.357	0.270	0.277	0.419	0.204	0.375	0.369	0.324
660	0.354	0.270	0.277	0.422	0.205	0.376	0.371	0.325
720	0.349	0.270	0.277	0.428	0.207	0.375	0.378	0.326

SUMMARY OF RESULTS

RMS ERROR OF LAST DAY UNDER DIFFERENT FILTER LENGTH

FILTER LENGTH	RMS ERROR OF LAST DAY							
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 7	MEAN
60	0.139	0.229	0.144	0.465	0.118	0.129	0.174	0.200
120	0.132	0.238	0.135	0.474	0.113	0.136	0.171	0.200
180	0.106	0.255	0.135	0.472	0.117	0.139	0.153	0.197
240	0.092	0.258	0.109	0.449	0.118	0.134	0.168	0.190
300	0.089	0.257	0.131	0.458	0.119	0.126	0.178	0.194
360	0.097	0.264	0.142	0.463	0.119	0.142	0.171	0.200
420	0.114	0.261	0.133	0.464	0.122	0.135	0.165	0.199
480	0.117	0.272	0.127	0.454	0.120	0.131	0.164	0.198
540	0.121	0.275	0.128	0.464	0.121	0.175	0.177	0.209
600	0.151	0.283	0.130	0.456	0.122	0.190	0.194	0.218
660	0.143	0.292	0.131	0.444	0.120	0.186	0.191	0.215
720	0.159	0.302	0.134	0.615	0.144	0.271	0.425	0.293

APPENDIX C

RESULTS OF SIX TEST RUNS OF DATA INTERPOLATION BY APPLYING SINGLE-CHANNEL PREDICTOR

DATA USED

1. Instrument Number 07

2. Instrument Location

Berkeley	California
City	State

Longitude: $122^{\circ}14'.1$ W

Latitude: $37^{\circ}52'.6$ N

3. Date of Data

April 22, 1971
April 30, 1971
May 8, 1971
May 15, 1971
May 24, 1971
May 30, 1971

April 22, 1971

ACTUAL DATA VALUE	FORWARD PREDICTION VALUE	FORWARD PREDICTION ERROR	BACKWARD PREDICTION VALUE	BACKWARD PREDICTION ERROR	AVERAGE PREDICTION VALUE	AVERAGE PREDICTION ERROR
34.950	34.996	-0.046	34.540	0.410	34.768	0.182
34.990	35.124	-0.134	34.631	0.359	34.877	0.113
35.100	35.304	-0.204	34.806	0.294	35.055	0.045
35.260	35.450	-0.190	34.993	0.267	35.221	0.038
35.400	35.522	-0.122	35.153	0.247	35.337	0.063
35.440	35.540	-0.100	35.257	0.183	35.398	0.042
35.380	35.502	-0.122	35.300	0.080	35.401	-0.021
35.280	35.444	-0.164	35.255	0.025	35.349	-0.069
35.100	35.364	-0.264	35.146	-0.046	35.255	-0.155
34.900	35.250	-0.350	34.994	-0.094	35.122	-0.222
34.700	35.159	-0.459	34.821	-0.121	34.990	-0.290
34.550	35.102	-0.552	34.674	-0.124	34.888	-0.338
34.450	35.100	-0.650	34.580	-0.130	34.840	-0.390
34.420	35.158	-0.738	34.565	-0.145	34.861	-0.441
34.470	35.247	-0.777	34.635	-0.165	34.941	-0.471
34.580	35.340	-0.760	34.769	-0.189	35.054	-0.474
34.730	35.418	-0.688	34.916	-0.186	35.167	-0.437
34.830	35.463	-0.633	35.056	-0.226	35.259	-0.429
34.850	35.404	-0.554	35.163	-0.313	35.283	-0.433
34.820	35.277	-0.457	35.221	-0.401	35.249	-0.429
34.720	35.141	-0.421	35.232	-0.512	35.186	-0.466
34.580	34.990	-0.410	35.190	-0.610	35.090	-0.510
34.450	34.894	-0.444	35.146	-0.696	35.020	-0.570
34.380	34.832	-0.452	35.137	-0.757	34.984	-0.604

RMS ERROR FOR FORWARD PREDICTION = 0.464

RMS ERROR FOR BACKWARD PREDICTION = 0.338

RMS ERROR FOR AVERAGE PREDICTION = 0.356

April 30, 1971

ACTUAL DATA VALUE	FORWARD PREDICTION VALUE	FORWARD PREDICTION ERROR	BACKWARD PREDICTION VALUE	BACKWARD PREDICTION ERROR	AVERAGE PREDICTION VALUE	AVERAGE PREDICTION ERROR
33.360	33.920	-0.560	32.860	0.500	33.390	-0.030
33.430	33.900	-0.470	33.000	0.430	33.450	-0.020
33.500	33.890	-0.390	33.100	0.400	33.495	0.005
33.510	33.890	-0.380	33.120	0.390	33.505	0.005
33.470	33.890	-0.420	33.120	0.350	33.505	-0.035
33.460	33.930	-0.470	33.120	0.340	33.525	-0.065
33.450	34.010	-0.560	33.080	0.370	33.545	-0.095
33.480	34.130	-0.650	33.040	0.440	33.585	-0.105
33.530	34.210	-0.680	33.010	0.520	33.610	-0.080
33.590	34.240	-0.650	33.000	0.590	33.620	-0.030
33.660	34.180	-0.520	33.020	0.640	33.600	0.060
33.690	34.080	-0.390	33.030	0.660	33.555	0.135
33.680	33.960	-0.280	33.070	0.610	33.515	0.165
33.590	33.770	-0.180	33.110	0.480	33.440	0.150
33.420	33.540	-0.120	33.130	0.290	33.335	0.085
33.250	33.350	-0.100	33.110	0.140	33.230	0.020
33.110	33.240	-0.130	33.010	0.100	33.125	-0.015
32.990	33.180	-0.190	32.870	0.120	33.025	-0.035
32.930	33.170	-0.240	32.760	0.170	32.965	-0.035
32.920	33.220	-0.300	32.650	0.270	32.935	-0.015
32.920	33.330	-0.410	32.610	0.310	32.970	-0.050
32.960	33.450	-0.490	32.590	0.370	33.020	-0.060
33.070	33.590	-0.520	32.620	0.450	33.105	-0.035
33.170	33.680	-0.510	32.710	0.460	33.195	-0.025

RMS ERROR FOR FORWARD PREDICTION = 0.435

RMS ERROR FOR BACKWARD PREDICTION = 0.421

RMS ERROR FOR AVERAGE PREDICTION = 0.072

May 8, 1971

ACTUAL DATA VALUE	FORWARD PREDICTION VALUE	ERROR	BACKWARD PREDICTION VALUE	ERROR	AVERAGE PREDICTION VALUE	ERROR
32.240	32.323	-0.083	32.006	0.234	32.164	0.076
32.270	32.392	-0.122	32.065	0.205	32.228	0.042
32.340	32.529	-0.189	32.150	0.190	32.339	0.001
32.450	32.618	-0.168	32.239	0.211	32.428	0.021
32.580	32.700	-0.120	32.320	0.260	32.510	0.070
32.660	32.789	-0.129	32.412	0.248	32.600	0.060
32.680	32.813	-0.133	32.478	0.202	32.645	0.035
32.650	32.803	-0.153	32.470	0.180	32.636	0.013
32.550	32.684	-0.134	32.333	0.217	32.508	0.042
32.410	32.485	-0.075	32.182	0.228	32.333	0.076
32.190	32.283	-0.093	32.019	0.171	32.151	0.039
32.020	32.155	-0.135	31.845	0.175	32.000	0.020
31.860	32.080	-0.221	31.687	0.173	31.883	-0.024
31.760	32.042	-0.282	31.594	0.166	31.818	-0.058
31.770	32.050	-0.280	31.580	0.190	31.815	-0.045
31.850	32.109	-0.259	31.621	0.229	31.865	-0.015
31.950	32.201	-0.251	31.685	0.265	31.943	0.007
32.030	32.287	-0.257	31.780	0.250	32.033	-0.003
32.080	32.341	-0.261	31.878	0.202	32.109	-0.030
32.120	32.369	-0.249	31.941	0.179	32.155	-0.035
32.100	32.350	-0.250	31.945	0.155	32.147	-0.048
32.060	32.301	-0.241	31.933	0.127	31.117	-0.057
32.010	32.220	-0.210	31.918	0.092	32.069	-0.059
31.960	32.145	-0.185	31.902	0.058	32.023	-0.063

RMS ERROR FOR FORWARD PREDICTION = 0.198

RMS ERROR FOR BACKWARD PREDICTION = 0.198

RMS ERROR FOR AVERAGE PREDICTION = 0,045

May 15, 1971

ACTUAL DATA VALUE	FORWARD PREDICTION VALUE	FORWARD PREDICTION ERROR	BACKWARD PREDICTION VALUE	BACKWARD PREDICTION ERROR	AVERAGE PREDICTION VALUE	AVERAGE PREDICTION ERROR
31.200	31.350	-0.150	31.440	-0.240	31.395	-0.195
31.200	31.350	-0.150	31.570	-0.370	31.460	-0.260
31.200	31.360	-0.160	31.670	-0.470	31.515	-0.315
31.190	31.390	-0.200	31.720	-0.530	31.555	-0.365
31.170	31.450	-0.280	31.720	-0.550	31.585	-0.415
31.170	31.530	-0.360	31.670	-0.500	31.600	-0.430
31.180	31.600	-0.420	31.590	-0.410	31.595	-0.415
31.230	31.660	-0.430	31.550	-0.320	31.605	-0.375
31.300	31.740	-0.440	31.550	-0.250	31.645	-0.345
31.360	31.770	-0.410	31.570	-0.210	31.670	-0.310
31.420	31.710	-0.290	31.630	-0.210	31.670	-0.250
31.430	31.570	-0.140	31.690	-0.260	31.630	-0.200
31.380	31.410	-0.030	31.720	-0.340	31.565	-0.185
31.280	31.230	0.050	31.730	-0.450	31.480	-0.200
31.100	31.040	0.060	31.680	-0.580	31.360	-0.260
30.920	30.890	0.030	31.550	-0.630	31.220	-0.300
30.740	30.760	-0.020	31.370	-0.630	31.065	-0.325
30.590	30.670	-0.080	31.230	-0.640	30.950	-0.360
30.500	30.680	-0.180	31.130	-0.630	30.905	-0.405
30.490	30.740	-0.250	31.020	-0.530	30.880	-0.390
30.520	30.840	-0.320	30.940	-0.420	30.890	-0.370
30.640	30.980	-0.340	30.940	-0.300	30.960	-0.320
30.810	31.110	-0.300	31.030	-0.220	31.070	-0.260
30.950	31.210	-0.260	31.180	-0.230	31.195	-0.245

RMS ERROR FOR FORWARD PREDICTION = 0.261

RMS ERROR FOR BACKWARD PREDICTION = 0.440

RMS ERROR FOR AVERAGE PREDICTION = 0.321

May 24, 1971

ACTUAL DATA VALUE	FORWARD PREDICTION VALUE	FORWARD PREDICTION ERROR	BACKWARD PREDICTION VALUE	BACKWARD PREDICTION ERROR	AVERAGE PREDICTION VALUE	AVERAGE PREDICTION ERROR
30.940	30.991	-0.051	31.013	-0.073	31.002	-0.062
30.930	31.019	-0.089	31.012	-0.082	31.015	-0.085
30.980	31.112	-0.132	31.028	-0.048	31.070	-0.090
31.060	31.243	-0.183	31.067	-0.007	31.155	-0.095
31.180	31.363	-0.183	31.154	0.026	31.258	-0.078
31.300	31.446	-0.146	31.265	0.035	31.355	-0.055
31.370	31.451	-0.081	31.367	0.003	31.409	-0.039
31.380	31.391	-0.011	31.452	-0.072	31.421	-0.041
31.340	31.274	0.066	31.430	-0.090	31.352	-0.012
31.230	31.090	0.140	31.315	-0.085	31.202	0.028
30.990	30.897	0.093	31.140	-0.150	31.018	-0.028
30.750	30.671	0.079	30.902	-0.152	30.786	-0.036
30.560	30.483	0.077	30.669	-0.109	30.576	-0.016
30.410	30.391	0.019	30.465	-0.055	30.428	-0.018
30.330	30.355	-0.025	30.316	0.014	30.335	-0.005
30.310	30.358	-0.048	30.251	0.059	30.304	0.005
30.340	30.420	-0.080	30.235	0.105	30.327	0.012
30.460	30.544	-0.084	30.294	0.166	30.419	0.041
30.640	30.682	-0.042	30.416	0.224	30.549	0.091
30.780	30.804	-0.024	30.570	0.210	30.687	0.093
30.880	30.891	-0.011	30.687	0.193	30.789	0.091
30.950	30.912	0.038	30.780	0.170	30.846	0.104
30.990	30.904	0.086	30.833	0.157	30.868	0.121
31.000	30.874	0.126	30.840	0.160	30.857	0.143

RMS ERROR FOR FORWARD PREDICTION = 0.094

RMS ERROR FOR BACKWARD PREDICTION = 0.121

RMS ERROR FOR AVERAGE PREDICTION = 0.070

May 30, 1971

ACTUAL DATA VALUE	FORWARD PREDICTION VALUE	FORWARD PREDICTION ERROR	BACKWARD PREDICTION VALUE	BACKWARD PREDICTION ERROR	AVERAGE PREDICTION VALUE	AVERAGE PREDICTION ERROR
30.300	30.390	-0.090	30.380	-0.080	30.385	-0.085
30.380	30.380	0.000	30.450	-0.070	30.415	-0.035
30.440	30.340	0.100	30.490	-0.050	30.415	0.025
30.470	30.320	0.150	30.510	-0.040	30.415	0.055
30.470	30.320	0.150	30.510	-0.040	30.415	0.055
30.450	30.340	0.110	30.460	-0.010	30.400	0.050
30.420	30.390	0.030	30.350	0.070	30.370	0.050
30.420	30.470	-0.050	30.250	0.170	30.360	0.060
30.440	30.550	-0.110	30.200	0.240	30.375	0.065
30.460	30.610	-0.150	30.190	0.270	30.400	0.060
30.530	30.640	-0.110	30.220	0.310	30.430	0.100
30.600	30.620	-0.020	30.260	0.340	30.440	0.160
30.630	30.530	0.100	30.330	0.300	30.430	0.200
30.620	30.380	0.240	30.380	0.240	30.380	0.240
30.540	30.200	0.340	30.420	0.120	30.310	0.230
30.380	30.040	0.340	30.450	-0.070	30.245	0.135
30.210	29.910	0.300	30.420	-0.210	30.165	0.045
30.050	29.860	0.190	30.340	-0.290	30.100	-0.050
29.930	29.840	0.090	30.260	-0.330	30.050	-0.120
29.900	29.900	0.000	30.170	-0.270	30.035	-0.135
29.920	30.010	-0.090	30.090	-0.170	30.050	-0.130
29.990	30.110	-0.120	30.080	-0.090	30.095	-0.105
30.080	30.190	-0.110	30.090	-0.010	30.140	-0.060
30.180	30.260	-0.080	30.140	0.040	30.200	-0.020

RMS ERROR FOR FORWARD PREDICTION = 0.158

RMS ERROR FOR BACKWARD PREDICTION = 0.195

RMS ERROR FOR AVERAGE PREDICTION = 0.113

BIBLIOGRAPHY

Clancy, E.P., The Tides, p. 1-87, p. 158-184, Anchor, 1969.

Gabel, R.A., and Roberts, R.A., Signals and Linear Systems, p. 1-205, Wiley, 1973.

Godin, G., The Analysis of Tides, p. 1-34, University of Toronto Press, 1972.

Hsu, H.P., Fourier Analysis, p. 53-160, Simon and Schuster, 1970.

Kinarimala, B., Kuo, F.F., and Tsao, N.K., Linear Circuits and Computation, p. 203-477, Wiley, 1973.

Maisel, L., Probability, Statistics and Random Processes, p. 183-256, Simon and Schuster, 1971.

Miller, K.S., Engineering Mathematics, p. 96-109, Dover, 1956.

Munk, W.H., and Cartwright, D.E., "Tidal Spectroscopy and Prediction," PHIL. TRANS. ROY. Soc. A, Vol. 259, p. 533-581, 1966.

Robinson, E.A., Statistical Communication and Detection, p. 103-282, Hafner, 1967.

Sakrison, D.J., Communication Theory, p. 9-130, p. 193-218, Wiley, 1968.

Spiegel, M.R., Advanced Mathematics, p. 242-257, McGraw-Hill, 1971.

Wyatt, S.P., Principles of Astronomy, p. 33-180, Allyn and Bacon, 1964.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameraon Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Department Chairman, Code 55 Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	2
4.	Chief of Naval Personnel PERS 11b Department of the Navy Washington, D.C., 20370	1
5.	Dr. M. D. Wood National Center for Earthquake Research 800 Menlo Avenue Menlo Park, California 94025	2
6.	Assoc. Professor R. H. Shudde, Code 55Su Department of Operations Research abd Administrative Sciences Naval Postgraduate School Monterey, California 93940	2
7.	Assoc. Professor D. R. Barr, Code 55Bn Department of Operations Research and Administrative Sciences Naval Postgraduate School Monterey, California 93940	2
8.	Capt. Chang, Chien-Wu, CN C/O Ms. Yu, Lian-Sheng Kuo-Kuang Middle School Tsoying, Taiwan, Rep. of China 813	2

Thesis 160675
C371 Chang
c.1 Single-channel
prediction for earth
tidal tilt data.